AN INNOVATIVE APPROACH TO THE ANALYSIS AND THE OPTIMIZATION OF 3D MAGNETIC FIELDS IN TOKAMAKS

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Potential causes of 3D field perturbations in tokamaks include inaccuracies and tolerances during the manufacture and assembly of the magnetic system. These deviations from the nominal configurations are known in literature as Error Fields (EFs) [1]. To counteract the EFs, a set of Error Field Correction Coils (EFCCs) can be used (Fig. 1). The Three Mode Error Index (TMEI) is one of the metrics used to quantify the impact of the EFs on the field map. The inaccuracies of the magnetic system are often difficult to estimate. Therefore, to identify the worst cases to be corrected and design of EFCC system needed to keep the TMEI value below a given threshold, a suitable stochastic analysis can be applied [1]. Further analyses will address two key areas: the impact of faults on the EFs correction capabilities [2] and the optimization of the EFCCs current to minimize energy consumption [3].

The activity also aims at deriving a linearized plasma response model for the analysis and the control of the 3D instabilities based on linear triangular finite elements. The formulation can also be used to determine the plasma reaction at steady state in case of 3D magnetic perturbations. The starting point is an innovative MHD equilibrium calculation with linear finite element triangles. This method, with a very limited computational effort, guarantees regularity of field derivatives, needed for the calculation of the terms of the linearized MHD model. The method is based on Helmholtz's theorem [4], using the finite element method (FEM) with linear shape functions for both $\partial\psi/\partial r$ and $\partial\psi/\partial z$ in the second order calculation. With the technique proposed in [5], the poloidal magnetic field can be determined by post-processing the standard FEM flux map as the solution to the minimization problem:

$$\min\left\{\int_\Omega \left[(\nabla\cdot\boldsymbol{B})^2 + \left(\nabla\times\boldsymbol{B} - \mu_0 J\boldsymbol{i_\phi}\right)^2\right] r^2 d\Omega + \gamma\int_{\partial\Omega}[\boldsymbol{n}\cdot\boldsymbol{B} - (\partial\psi/\partial t)/r)]^2 r d\Gamma\right\}$$

where γ is a suitable non-dimensional factor, whereas $J = J(r,\psi)$ and $\partial\psi/\partial t$ (the tangential component of $\nabla\psi$) are taken from the standard FEM solution. The procedure can also be applied to magnetostatic calculations in the presence of magnetic media [6]. The method has also been applied to accurately evaluate the plasma boundary and field lines in the scrape-off layer, particularly near the X-point [7, 8]. This is of the utmost importance for the power exhaust issues in tokamaks (Fig. 2). For a given accuracy, the computational cost of the procedure is significantly lower than alternative methods relying on finer first order discretization or techniques using triangular C^1 finite elements.

The linearization approach was initially developed for 2D axisymmetric configurations. The focus of this study is to determine the variation of the plasma configuration due to minor alterations of the equilibrium external currents, or, equivalently, slight changes of the external poloidal flux $\delta\psi_{ext}$. The procedure, based on FEM, starts with the solution of the *fixed boundary problem* (i.e., the 2nd order non-linear PDE *Grad-Shafranov equation*) inside the prescribed plasma region Ω_{pl}. The *degrees of freedom* of the problem are the poloidal flux per radian ψ_b at the boundary $\partial\Omega_{pl}$ and a set of current density shape parameters $\boldsymbol{\alpha}$. Subsequently, the linearised flux response in the plasma region is computed by differentiating the PDE equation in Ω_{pl} and evaluating the variation of poloidal flux due to small perturbations of the degrees of freedom $\delta\psi_b$ and $\delta\boldsymbol{\alpha}$. Finally, additional output quantities are calculated in the post-processing phase. Since the mesh is required only in the plasma region, the procedure is considerably more flexible and lighter in terms of computational effort than models based on *free boundary problems* [9]. Moreover, thanks to the accurate magnetic field line tracking [5, 6, 7], a reliable estimation of the variation of plasma-wall gaps and plasma deformation can be obtained, as shown in Fig. 3. Finally, the model is applicable for dynamic analysis (evaluation of growth rate, growth modes, dynamic responses of measurements etc.).

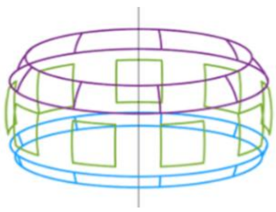

Fig. 1: Sketch of an EFCCs system [1].

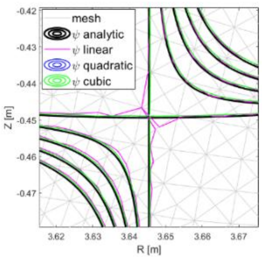

Fig. 2: Flux lines [7-8] obtained with standard (linear) and innovative (quadratic or cubic) procedure in the vicinity of the lower X-point.

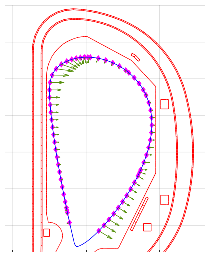

Fig. 3: Plasma displacement due to the variation of a PF coil current.

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