GAME THEORY AND NETWORK MODELS FOR THE RECONSTRUCTION OF ARCHAEOLOGICAL NETWORKS *

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ABSTRACT: Most models for the reconstruction of networks in archaeology are based largely on the assumption of dyadic independence. Here we propose to use Exponential Random Graph Models (ERGMs) and concepts from game-theory to account for tie dependence while reconstructing networks in the past. To illustrate the applicability of this approach we consider pre-colonial (AD 100-400) Caribbean networks.

KEYWORDS: network models, game theory, stable equilibria, archaeology

1 Archaeology and Networks

Network approaches have been adopted in many disciplines to analyse relational data. Most of the applications of network analysis seek to describe, model, and test hypotheses concerning the structure of (a sequence of) observed networks, and determine either how the structure evolves or what the implications of such a structure are on some outcome variables.

The application of network analysis in archaeology does not fall into this framework since networks of interaction between past cultures and communities cannot be observed completely. In fact, archaeological evidence (e.g., artefact counts) and historical documentation (e.g., manuscripts) are fragmentary and do not provide enough information to get anywhere near of a complete pictures of networks in the past. Investigating such networks is relevant to archaeological research, however, because patterns of interaction can contribute to the explanation of various phenomena such as the spatial distribution of artefacts, migrations, or the emergence of settlement hubs. Developing methods to reconstruct archaeological networks and to infer unobserved data is therefore

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crucial in such a scenario. Here we propose to use Exponential Random Graph Models and concepts from game-theory to reconstruct networks in the past.

In the following, we refer to an archaeological network as a network where nodes are archaeological sites, providing evidence to the existence of settlements characterized by their geographical location and size, and ties represent (the volume of) interactions between them.

### 2 Reconstructing archaeological networks

Several deterministic and model-based approaches have been proposed in the past decades, starting from a number of pioneering studies in the 1960s and 70s. Irrespective of the method used, the reconstruction of networks in the past necessitates the formulation of hypotheses regarding the principles of interaction. Two prominent assumptions have been used.

The first states that the geographical space in which sites are located plays a major role in shaping networks of interaction. In fact, given the limitations of travel technology in the past, reconstruction methods assumed that interactions between sites close to each other are more likely than those between far away sites. Maximum Distance Networks and Proximal Point Analysis (Terrell, 1977) represent the simplest models built on this assumption. They are deterministic models resulting in a fixed binary network, where the existence of a tie is determined by an indicator function depending on distance.

The second assumption is related to the size of a site. In particular, it is assumed that bigger sites are able to sustain more relationships than smaller sites. Gravity models (Hodder, 1974) and the ariadne model (Knappett et al., 2008) make use of this assumption, beyond that on distance, to reconstruct archaeological networks. While the gravity model results in a fixed weighted network, the ariadne model yields weighted networks corresponding to the local minima of a cost-benefit function evaluated on all the possible configurations defined on the set of sites.

These assumptions and the corresponding models are too simplistic since the presence and the strength of ties depend only on exogenous variables (distance and attributes of sites), and the constraints that are imposed on them. Thus, interactions are determined solely on the basis of dyadic elements, without accounting for tie dependence. This dependency is one of the crucial points in network science and implies that the existence of a tie may affect the existence of other ties. Often, the global configuration of a network (macro-level) is assumed to be the result of micro-level mechanisms governing the formation and dissolution of ties. In the present case this amounts to modelling the
decision of sites to create, maintain and terminate ties.

In our opinion, disregarding micro-level mechanisms and tie dependency in reconstructing archaeological networks are the main limitations of previous models. Therefore, we propose to apply statistical models for the analysis of networks and game-theoretic concepts to overcome such limitations.

3 Network models for reconstructing archaeological networks

Recent work (Amati et al., in preparation) has proposed the use of network models, and in particular Exponential Random Graph models (ERGMs), to account for dependencies within ties, and between ties and node attributes, when reconstructing archaeological networks.

ERGMs (Robins et al., 2007) are stochastic models for the analysis of cross-sectional data, that is a network observed at one single point in time. They assume that the configuration of an observed network results from the combination of patterns of ties corresponding to the local mechanisms that could have generated the observed network.

Formally, ERGMs are a family of probability distributions over the set $X$ of all possible networks on a fixed set of nodes. The general form of ERGMs is given by

$$P(X = x) = \frac{1}{\kappa(\theta)} \exp \left( \sum_{k=1}^{K} \theta_k s_k(x) \right)$$

where $s_k(x)$ are network statistics, $\theta_k$ are the corresponding parameters, and $\kappa(\theta)$ is a normalizing constant. The distribution in (1) is derived by the application of the Hammersley-Clifford Theorem stating which statistics are allowed to be used given a certain dependence assumption. A statistic $s_k(x)$ counts the number of local configurations of type $k$ in the network $x$, and it is specified on the grounds of theories concerning the micro-mechanisms that might have lead to the observed network. Therefore, the assumptions characterizing the models introduced in the previous section (and many other assumptions) can be incorporated into an ERGM by means of the set of statistics $s_k(x)$. The corresponding parameter $\theta_k$ measures the importance of the configuration of type $k$ in explaining the global network configuration.

Given a specification of an ERGM and values for the parameters $\theta$, it is possible to sample (binary) random networks from (1) using Markov Chain Monte Carlo methods. It follows that, when archaeological assumptions regulating the occurrence of interaction can be translated into the statistics $s_k(x)$, we can specify an ERGM, and generate networks whose structure is coherent.
with these assumptions. The sample networks are characterized by their probability, and the most likely simulated networks may be assumed to be more plausible reconstructions of a network in the past.

4 A game-theoretic approach

Compared to the models suggested in the archaeological literature, ERGMs provide a flexible framework to reconstruct archaeological networks by simultaneously accounting for assumptions on distance, site sizes and other nodal attributes, as well as on dependencies within ties. However, like those models, ERGMs miss a micro-foundation explaining how the global configuration of a network results from the decisions of sites on creating, terminating and maintaining ties.

To overcome this limitation, a cost-benefit approach is adopted here. In particular, we use concepts from game theory to relate ERGMs to the outcome of a dynamic strategic game in which the interaction framework is designed as a potential game and sites take decisions in an attempt to maximize their utility, albeit with error. After reviewing the relationship between site utility and network statistics, we characterize the archaeological networks resulting from the sampling from ERGMs in terms of socially desirable equilibria and strategic stability. We suggest to use stable networks as plausible reconstructions of a network in the past instead of using the criterion based on the most likely sampled networks. To illustrate the applicability of this approach we consider pre-colonial (period AD 100-400) Caribbean networks.

References


