III Workshop on Trends in Nonlinear Analysis

In honor of Stella Piro-Vernier's 70th birthday

Cagliari, September, 7-9, 2017

Contents

1 INTRODUCTION AND ACKNOWLEDGMENTS

We are delighted to be hosting the III WORKSHOP ON TRENDS IN NONLINEAR ANALYSIS here at the University of Cagliari (Italy). The aim of such an activity is to gather well-established experts and young researchers, all specialized on Partial Differential Equations and related topics, and let them share their results and discuss further developments and open problems. Precisely, the activity is aimed at discussing issues linked to the following fields:

- Analysis and Applications of Ordinary and Partial Differential Equations,
- Variational Methods for the Resolution of Boundary Value Problems,
- Calculus of Variations and Non-Linear Problems.

Mainly, this meeting was possible thanks to the generous availability of all the speakers who participated in the activity. Moreover, we would like to express our gratitude to the *Dipartimento di Matematica ed Informatica dell'Università degli Studi di Cagliari* and the *INdAM -Istituto Nazionale di Alta Matematica "F. Severi" (GNAMPA)*; without the support of these sponsors the conference would have been not possible. Finally, we also appreciate the effort of all the other colleagues who, directly or indirectly, offered their useful contribution in the organization of this Workshop, and we thank Valentina Formica for the nice poster picture.

Cagliari, 7 September 2017

The Scientific Committee

Antonio Iannizzotto (*Università degli Studi di Cagliari*)
Monica Marras (*Università degli Studi di Cagliari*)
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2 INVITED SPEAKERS

1. VERENA BÖGELEIN (UNIVERSITÄT ERLANGEN-NÜRNBERG)
2. PASQUALE CANDITO (UNIVERSITÀ MEDITERRANEA DI REGGIO CALABRIA)
3. GIOVANNI CUPINI (UNIVERSITÀ DI BOLOGNA)
4. FRANK DUZAAR (UNIVERSITÄT ERLANGEN-NÜRNBERG)
5. MATTEO FOCARDI (UNIVERSITÀ DEGLI STUDI DI FIRENZE)
6. SALVATORE A. MARANO (UNIVERSITÀ DEGLI STUDI DI CATANIA)
7. PAOLO MARCELLINI (UNIVERSITÀ DEGLI STUDI DI FIRENZE)
8. GÉRARD PHILIPPIN (UNIVERSITÉ LAVAL, QUEBEC)
9. SERGIO POLIDORO (UNIVERSITÀ DEGLI STUDI DI MODENA E REGGIO EMILIA)
10. GIOVANNI PORRU (UNIVERSITÀ DEGLI STUDI DI CAGLIARI)
11. PATRIZIA PUCCI (UNIVERSITÀ DEGLI STUDI DI PERUGIA)
12. MARCO SQUASSINA (UNIVERSITÀ CATTOLICA DEL SACRO CUORE)
13. ENRICO VALDINOCI (UNIVERSITÀ DEGLI STUDI DI MILANO)
14. VINCENZO VESPRI (UNIVERSITÀ DEGLI STUDI DI FIRENZE)
15. ANTONIO VITOLO (UNIVERSITÀ DEGLI STUDI DI SALERNO)

Existence of variational solutions in non-cylindrical domains

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Abstract

In this talk we establish the existence of evolutionary variational solutions in non-cylindrical domains. Our model problem is a gradient flow of the form

$$\partial_t u - \operatorname{div} Df(Du) = 0$$

in a non-cylindrical bounded domain $E \subset \mathbb{R}^n \times [0,T]$ with zero Dirichlet boundary data. For the integrand f we only assume that it is coercive and convex. We prove the existence of variational solutions. The only assumption on the domain E is that it has zero boundary measure. In some special situations, for instance in the case of non-decreasing domains, we can show uniqueness of variational solutions and the existence of the time derivative in L^2. This is joint work with Frank Duzaar (Erlangen), Christoph Scheven (Duisburg-Essen), and Thomas Singer (Erlangen).

A coincidence point theorem and its applications

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Abstract

The aim of this talk is to present a coincidence point theorem for sequentially weakly continuous maps. Moreover, as a consequence, an existence result for differential inclusions is pointed out. Finally, some applications to the study of different nonlinear boundary value problems are showed.

References

[1] G. Bonanno, P. Candito, D. Motreanu, *A coincidence point theorem for sequentially continuous mappings*, J. Math. Anal. Appl. **435** (2016), 606-615.

[2] P. Candito, R. Livrea, *An existence result for a Neumann problem*, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal. **22** (2015), 481-488.

Local boundedness of vectorial minimizers via De Giorgi's method

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Abstract

A recent result concerning the local boundedness of vectorial minimizers of a suitable class of polyconvex functionals will be discussed ([1]). The proof relies on the powerful and elegant method introduced by De Giorgi in his fundamental paper (1957).

References

[1] G. Cupini, F. Leonetti, E. Mascolo, *Local boundedness for minimizers of some polyconvex integrals*, Arch. Ration. Mech. Anal. **224**, pp. 269–289

A variational approach to the porous medium equation

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Abstract

In this talk we establish an existence theory for the porous medium equation

$$\partial_t u^m - \Delta u = 0,$$

and more generally, for doubly nonlinear evolution equations of the type

$$\partial_t b(u) - \operatorname{div}\big(Df(Du)\big) = 0,$$

where f is only assumed to be coercive and convex. Our approach is purely variational and inspired by minimizing movements. It is flexible enough to deal also with obstacle problems with low regularity of the obstacle or time dependent boundary data. This is joint work with Verena Bögelein (Salzburg), Paolo Marcellini (Florence), and Christoph Scheven (Duisburg-Essen).

The measure and the structure of the free boundary in the lower dimensional obstacle problem

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Abstract

In this talk I present the main results of a recent paper in collaboration with E. Spadaro (Uni. Leipzig) on the regularity of the free boundary for a class of lower dimensional obstacle problems, including the classical scalar Signorini problem. We prove the first results concerning the global structure of the free boundary, in particular showing its local finiteness and its rectifiability.

References

[1] M. Focardi, E. Spadaro *On the measure and the structure of the free boundary of the fractional obstacle problem.*, Preprint arXiv:1703.00678.

Some recent results on the Dirichlet problem for (p,q)-Laplace equations

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Abstract

Let Ω be a bounded domain in $\mathbb{R}^N$ with a C^2-boundary $\partial\Omega$, let $1 < q \le p < N$, and let $\mu \in \mathbb{R}_0^+$. Consider the Dirichlet problem

$$-\Delta_p u - \mu\Delta_q u = g(x,u) \quad \text{in} \quad \Omega, \qquad u = 0 \quad \text{on} \quad \partial\Omega, \tag{1}$$

where Δ_r, $r > 1$, denotes the r-Laplacian, namely

$$\Delta_r u := \operatorname{div}(|\nabla u|^{r-2}\nabla u) \quad \forall u \in W_0^{1,r}(\Omega),$$

$p = q$ iff $\mu = 0$, while $g : \Omega \times \mathbb{R} \to \mathbb{R}$ satisfies Carathéodory's conditions. The non-homogeneous differential operator $Au := \Delta_p u + \mu\Delta_q u$ that appears in (1) is usually called (p,q)-Laplacian. It stems from a wide range of important applications, including bio-physics, plasma physics, reaction-diffusion equations, etc. That's why the relevant literature looks daily increasing and numerous meaningful papers on this subject are by now available.

This talk, chiefly based on the survey [1], provides a short account of some recent existence and multiplicity results involving (1), often with a precise sign information. To

avoid technicalities, special but significant cases of more general theorems are stated. So, the function g is supposed purely autonomous as well as of the type

$$g(x,t) := \alpha|t|^{p-2}t + \beta|t|^{q-2}t + f(t), \quad (x,t) \in \Omega \times \mathbb{R},$$

where the perturbation f lies in $C^1(\mathbb{R})$ and exhibits a suitable growth rate near $\pm\infty$ and/or zero. Results are diversified according to the asymptotic behavior at infinity, frequently under the simplified condition

$$-\infty < \lim_{|t|\to+\infty} \frac{f(t)}{|t|^{r-2}t} < +\infty,$$

with appropriate $r > 1$.

References

[1] Salvatore A. Marano and Sunra J.N. Mosconi, *Some recent results on the Dirichlet problem for (p,q)-Laplace equations*, Discrete Contin. Dyn. Syst. Ser. S, in press.

Regularity and existence for elliptic and parabolic equations and systems under general and p,q growth conditions

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Abstract

We give some recent *existence* and *interior regularity results* - partly obtained in collaboration with *Giovanni Cupini, Michela Eleuteri* and *Elvira Mascolo* - for elliptic partial differential *equations* in divergence form, or elliptic *systems* of m partial differential equations in divergence form of the type

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} a_\alpha^i (x, u(x), Du(x)) = b_\alpha (x, u(x), Du(x)), \quad \alpha = 1,2,\dots,m,$$

for maps $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$. Here the vector field $\left(a_\alpha^i (x,s,\xi)\right)$ assumes values in the set of $m \times n$ matrices and it satisfies some *general growth conditions* with respect to the gradient variable $\xi \in \mathbb{R}^{m\times n}$, sometine *$p,q$ growth conditions*.

As a part of a joint research-project started in 2013 with *Verena Bögelein and Frank Duzaar*, we consider the evolution problem associated with a convex integrand $f\colon \mathbb{R}^{m\times n} \to [0,\infty)$ satisfying - for instance - some p,q-growth assumption. To establish the existence of solutions we introduce the concept of *variational solutions*. In contrast to weak solutions, i.e. mappings $u\colon \Omega_T \subset \mathbb{R}^{n+1} \to \mathbb{R}^m$ which solve

$$\partial_t u - \mathrm{div} Df(Du) = 0$$

weakly in Ω_T, variational solutions in general exist under a much weaker assumption on the gap $q-p$.

In particular, if $2 \le p \le q < p + \min\left\{1, \frac{4}{n}\right\}$, we obtain the existence of variational solutions and we also show that they are actually - in this case - weak solutions. This

means that any solution u automatically admits the necessary higher integrability of the spatial derivative Du to satisfy the parabolic system in the weak sense, i.e. we prove that

$$u \in L^q_{\mathrm{loc}}\left(0,T;W^{1,q}_{\mathrm{loc}}(\Omega,\mathbb{R}^m)\right).$$

Old and new results on eigenvalues of the vibrating membrane problem

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Abstract

A number of isoperimetric inequalities derived 50 years ago by G. Szegö and J. Hersch will be reviewed. Some recent extensions will be presented.

Harnack Inequalities for Hypoelliptic Second Order Partial Differential Equations

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Abstract

We consider second order hypoelliptic operators of the form

$$\mathscr{L}u(x,t) := \sum_{j=1}^{m} X_j^2 u(x,t) + X_0 u(x,t) - \partial_t u(x,t), \qquad (x,t) \in \Omega \subset \mathbb{R}^{N+1},$$

where Ω is a bounded open subset of $\mathbb{R}^{N+1}$ and $X_0 - \partial_t, X_1, \dots, X_m$ are smooth vector fields satisfying the Hörmander's condition. We give a geometric sufficient condition on the point $(x_0,t_0) \in \Omega$ and on the compact sets $K \subset \Omega$ for which the Harnack inequality

$$\sup_K u \le C_K u(x_0,t_0),$$

holds for all non-negative solutions u to the equation $\mathscr{L}u = 0$ in Ω.

This result has been proven in [1] under the further assumption that the vector fields $X_0 - \partial_t, X_1, \dots, X_m$ are invariant with respect to a suitable translation group on $\mathbb{R}^{N+1}$. This last condition has been removed in [2].

References

[1] C. Cinti, K. Nyström, S.Polidoro, *A Note on Harnack Inequalities and Propagation Sets for a Class of Hypoelliptic Operators*, Potential Analysis **33**(4) (2010), pp. 341–354,

[2] A. E. Kogoj, S.Polidoro, *Harnack Inequality for Hypoelliptic Second Order Partial Differential Operators*, Potential Analysis **35**(3) (2016), pp. 545–555.

Optimization problems in classes of rearrangements

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Abstract

This talk is concerned with maximization and minimization of a functional associated with the solution of a partial differential equation depending on functions which belong to a class of rearrangements. We prove existence and uniqueness results, and present the configuration of optimal solutions.

References

[1] Monica Marras, Giovanni Porru, Stella Vernier-Piro, *Optimization problems for eigenvalues of p-Laplace equations*, Journal of Mathematical Analysis and Applications **398** (2013), 766–775

Degenerate Kirchhoff wave problems involving nonlocal integro–differential operators

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Abstract

A great attention has been drawn to the study of fractional and nonlocal problems of Kirchhoff type, since they arise in a quite natural way in many different applications. In the first part of the talk we first present recent results concerning wave Kirchhoff problem driven by a nonlocal integro–differential operator, as global existence (even under critical initial conditions), vacuum isolating and blow up of solutions. The proof arguments combine the Galerkin method with the potential well theory, see [1].

The second part of the talk deals with the question of global and local asymptotic stability for nonlinear damped Kirchhoff systems, with homogeneous Dirichlet boundary, under fairly natural assumptions on the external force $f = f(t,x,u)$ and the distributed damping $Q = Q(t,x,u,u_t)$. In [2] the Kirchhoff coefficient represents a nonlocal dissipative

effect in the dynamical damping term, which models a frictional mechanism acting on the body.

All the results extend in several directions recent theorems and cover also the so–called *degenerate case*, that is the case in which M is zero at zero. From a physical point of view, this represents the fact that the base tension of the string modeled by the equation is zero: a very realistic condition. To overcome the difficulties due to the degeneracy of the models we have to make use of different approaches. The conclusions also raise, and leave open, a number of other intriguing questions, which are briefly presented.

References

[1] N. Pan, P. Pucci and B. Zhang, *Degenerate Kirchhoff–type hyperbolic problems involving the fractional Laplacian*, submitted for publication, pages 25.

[2] P. Pucci and S. Saldi, *Asymptotic stability for nonlinear damped Kirchhoff systems involving the fractional p–Laplacian operator*, J. Differential Equations (2017), pages 43 DOI:10.1016/j.jde.2017.02.039

Approximation results for magnetic Sobolev norms

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Abstract

We discuss recent results [6, 4, 5] on the approximation of classical magnetic Sobolev norms with nonlocal (convex and nonconvex) magnetic energies. In the convex case, our results extend the so called Bourgain-Brezis-Mironescu limit to the magnetic setting. We also present some existence results for fractional problems involving an external magnetic field.

References

[1] P. d'Avenia, M. Squassina, *Ground states for fractional magnetic operators*, ESAIM Control Optim. Calc. Var. (2017), to appear.

[2] X. Mingqui, P. Pucci, M. Squassina, B. Zhang, *Nonlocal Schrodinger-Kirchhoff equations with external magnetic field*, Discrete Contin. Dyn. Syst. A **37** (2017), 503–521.

[3] A. Pinamonti, M. Squassina, E. Vecchi, *The Maz'ya-Shaposhnikova limit in the magnetic setting*, J. Math. Anal. Appl. **449** (2017), 1152–1159.

[4] A. Pinamonti, M. Squassina, E. Vecchi, *Magnetic BV functions and the Bourgain-Brezis-Mironescu formula*, Adv. Calc. Var. (2017), to appear.

[5] H.-M. Nguyen, A. Pinamonti, M. Squassina, E. Vecchi, *New characterization of magnetic Sobolev spaces*, (2017) preprint.

[6] M. Squassina, V. Volzone, *Bourgain-Brezis-Mironescu formula for magnetic operators*, C. R. Math. Acad. Sci. Paris **354** (2016), 825–831.

[7] B. Zhang, M. Squassina, X. Zhang, *Fractional NLS equations with magnetic field, critical frequency and critical growth*, Manuscripta Math. (2017), to appear.

Nonlocal minimal surfaces

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Abstract

Nonlocal minimal surfaces, as introduced in [3], are objects which resemble hypersurfaces of minimal perimeter when the parameter approaches the integer value, but present very special – and sometimes unexpected – characteristics. We discuss some recent results concerning interior regularity [10, 4, 1], Bernstein theorems [9, 8], perimeter estimates [5] and boundary stickiness [6, 7, 2].

References

[1] B. Barrios, A. Figalli, E. Valdinoci, *Bootstrap regularity for integro-differential operators and its application to nonlocal minimal surfaces*, Ann. Sc. Norm. Super. Pisa Cl. Sci. (5) **13**(3) (2014), pp. 609–639.

[2] C. Bucur, L. Lombardini, E. Valdinoci, *Complete stickiness of nonlocal minimal surfaces for small values of the fractional parameter*, arXiv:1612.08295.

[3] L. Caffarelli, J.-M. Roquejoffre, O. Savin, *Nonlocal minimal surfaces*, Comm. Pure Appl. Math. **63**(9) (2010), pp. 1111–1144.

[4] L. Caffarelli, E. Valdinoci, *Regularity properties of nonlocal minimal surfaces via limiting arguments*, Adv. Math. **248** (2013), pp. 843–871.

[5] E. Cinti, J. Serra, E. Valdinoci, *Quantitative flatness results and BV-estimates for stable nonlocal minimal surfaces*, arXiv:1602.00540.

[6] S. Dipierro, O. Savin, E. Valdinoci, *Boundary behavior of nonlocal minimal surfaces*, J. Funct. Anal. **272**(5) (2017), no. 5, pp. 1791–1851.

[7] S. Dipierro, O. Savin, E. Valdinoci, *Graph properties for nonlocal minimal surfaces*, Calc. Var. Partial Differential Equations **55**(4) (2016), Paper No. 86, 25 pp.

[8] A. Farina, E. Valdinoci, *Flatness results for nonlocal minimal cones and subgraphs*, arXiv:1706.05701.

[9] A. Figalli, E. Valdinoci, *Regularity and Bernstein-type results for nonlocal minimal surfaces*, to appear in J. Reine Angew. Math.

[10] O. Savin, E. Valdinoci, *Regularity of nonlocal minimal cones in dimension* 2, Calc. Var. Partial Differential Equations **48**(1-2) (2013), pp. 33–39.

Pointwise estimates for a class of degenerate parabolic equations

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Abstract

We consider the Cauchy problem associated to a class of nonlinear degenerate parabolic equations, whose prototype is the parabolic p-Laplacian ($\frac{2N}{N+1} < p < \infty$). In his seminal paper, after stating the Harnack estimates, Moser proved almost optimal estimates for the parabolic kernel by using the so called Harnack chain method. In the linear case sharp estimates come by using Nash's approach. Fabes and Stroock proved that Gaussian estimates are equivalent to a parabolic Harnack inequality. In several papers in collaboration with Bögelein, Calahorrano, Piro-Vernier and Ragnedda, by using the DiBenedetto De Giorgi approach we prove optimal kernel estimates for quasilinear parabolic equations. Lastly we use these results to prove existence and sharp pointwise estimates from above and from below for the fundamental solutions.

Fully nonlinear elliptic equations with Keller-Osserman absorption terms

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Abstract

The aim of the talk is to report on recent results about existence, uniqueness and qualitative properties of boundary blow-up solutions of fully nonlinear elliptic equations which describe a stationary diffusion process due to a superlinear reaction term. Classically, the existence of large solutions, in a bounded domain, for the semilinear equation $\Delta u = f(u)$ with $f \geq 0$ depends on the growth rate of the function $f(t)$ as $t \to \infty$, according to the well known Keller-Osserman condition

$$\int_1^\infty \frac{ds}{\sqrt{F(s)}} < \infty,$$

where $F(s)$ is the antiderivative of $f(s)$ vanishing at $s = 0$.
Here the equation $H(x,u,Du,D^2u) = f(u)$ for a fully nonlinear second-order elliptic operator H will be considered generalizing the results of the linear case as well as new conditions on the zero order term f will be discussed, which seem to improve the classical results also when the principal term is the Laplace operator.
On the other hand, the failure of the above Keller-Osserman condition provides the existence of entire subsolutions: $\Delta u \geq f(u)$ in all $\mathbb{R}^n$. A fully nonlinear, possibly degenerate, elliptic counterpart of this result will also be illustrated.

4 PROGRAM OF THE MEETING

All the invited speakers will present their contributions at the room “Alfa” of the Faculty of Engineering and Architecture of the University of Cagliari (via Marengo 2, Cagliari), and according to the following program.

	September, 7	September, 8	September, 9
		Morning Session	
Chair		*Polidoro*	*Ragnedda*
9:00 - 9:30		Marcellini	Valdinoci
9:40 - 10:10		Focardi	Polidoro
10:10 - 10:50		**Coffee break**	**Coffee break**
10:50 - 11:20		Cupini	Squassina
11:30 - 12:00		Candito	Pucci
		Afternoon Session	
Chair	*Porru*	*Pucci*	
15:00 - 15:30	**Opening**	Philippin	
15:40 - 16:10	Vespri	Vitolo	
16:10 - 16:50	**Coffee break**	**Coffee break**	
16:50 - 17:20	Duzaar	Marano	
17:30 - 18:00	Bögelein	Porru	
20:30		**Social Dinner**	